The University of Nottingham

DEPARTMENT OF MECHANICAL, MATERIALS, AND MANUFACTURING

A LEVEL 2 MODULE, SPRING SEMESTER 2014-2015

MECHANICS OF SOLIDS 3

Time allowed TWO Hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced

Answer any FOUR questions

Only silent, self contained calculators with a Single-Line Display or Dual-Line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so

In this exam students are required to answer 4 out of 6 questions. If students answer more than the required number of questions only the required number will be marked, so only the first 4 solutions will be marked.

If students attempt additional questions they should clearly indicate on their script which solutions they want to be marked - simply putting a line through solutions that should be disregarded is recommended.

ADDITIONAL MATERIAL: None

INFORMATION FOR INVIGILATORS:

Question papers should be collected in at the end of the exam – do not allow candidates to take copies from the exam room.

Turn Over

1. A rigid beam supported by a truss pin-jointed structure, as shown in Figure Q1, is subjected to a uniformly distributed load w acting on the full length of the beam span

The truss members are made from solid circular cross-section steel, with a modulus of elasticity E = 200GPa and have a diameter of 40mm.

(a) Indicate the most critical member due to buckling and give the reason supported by calculations. [15 marks]

(b) By taking a safety factor of 2, calculate the maximum value of the external load w in N/m that the structure is allowed to carry. [10 marks]

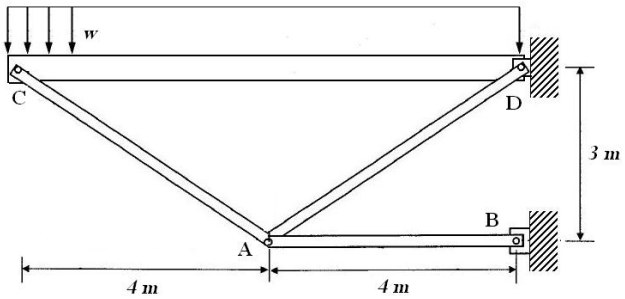

Figure Q1

2. Figure Q2 shows the cross-section of a beam.

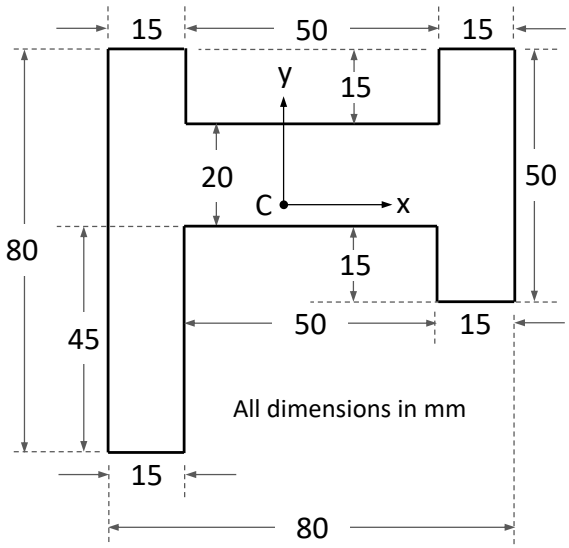

Figure Q2

Determine:

(a) The position i.e. x and y coordinates, of the Centroid of Area, C. [6 marks]

(b) The Principal 2nd Moments of Area. [13 marks]

(c) The orientation of the Principal Axes with respect to the x-y co-ordinate system (show on a sketch of the cross-section). [6 marks]

3. (a) Derive an expression for the strain energy in a uniform section beam of length, L, with Young's Modulus, E, and second moment of area, I, subjected to a bending moment, M. [5 marks]

(b) The structure shown in Figure Q3 is made up of a straight horizontal section of length, L_1, of 250mm, which is free at one end and attached to a straight vertical section of length, L_2, of 500mm, which is built into the ground at the other end, as shown in Figure Q3. A vertical point load, P, of 1kN, is applied at the free tip of the horizontal section as shown in the figure. Length L_1 has a symmetrical, hollow, square cross-section with outer and inner dimensions of 50mm and 20mm and length L_2 has a solid square cross-section with an outer dimension of 50mm. The structure is made of aluminium with a Young's Modulus, E, of 70GPa.

L_1 = 250mm L_2 = 500mm P = 1kN E = 70GPa

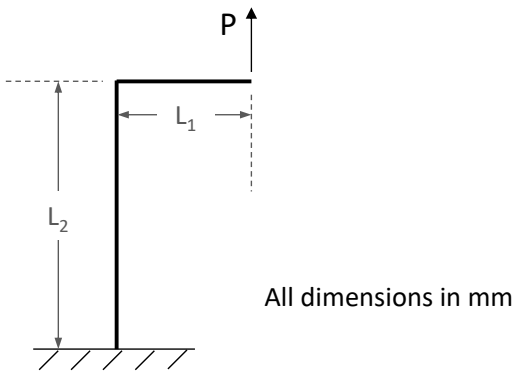

Figure Q3

Determine the horizontal deflection at the tip of the beam (i.e. at the position of the applied load, P). [20 marks]

Turn Over

4. A long unrestrained closed end cylinder made from steel with an inner radius of 300mm and an outer radius of 305mm is subjected to an internal pressure of 1MN/m^2 and a temperature difference between the inside and outside of the wall. The temperature of the inside is 4^{0}C higher than the outside. The distribution of temperature across the wall thickness is considered linear and the thermal radial stress is negligible.

The thermal expansion coefficient of the steel is 12 x 10^{-6}/degree and the Poisson's ratio is 0.3.

(a) Calculate the axial stress of the cylinder at the inside and outside of the wall. [9 marks]

(b) Calculate the hoop stress at the inside and outside of the wall. [8 marks]

(c) Determine the location and the magnitude of the maximum shear stress. If the yield point of the material is 250MPa, calculate the safety factor against yielding. [8 marks]

Lame's equations:

$$\sigma_r = A - \frac{B}{r^2} \qquad \sigma_\theta = A + \frac{B}{r^2}$$

Stress-strain relationship:

$$\varepsilon_\theta = \frac{1}{E}\left(\sigma_\theta - \nu\left(\sigma_r + \sigma_z\right)\right) + \alpha T$$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu\left(\sigma_r + \sigma_\theta\right)\right) + \alpha T$$

5. The section shown in Figure Q5 carries a shear force, S = 40kN down the vertical centre line.

(a) Determine the position of the centroid, G, from the base of the section X-X. [4 marks]

(b) Calculate the second moment of area of the section about the neutral axis (N.A.). [5 marks]

(c) Determine the vertical shear stress at points A, B, C and G. [12 marks]

(d) Sketch the vertical shear stress distribution along the vertical centre line of the section. [4 marks]

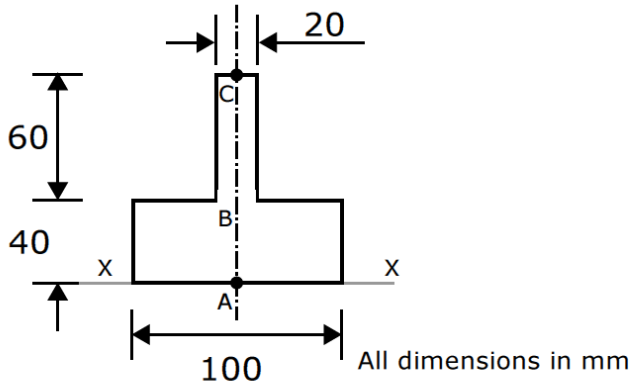

Figure Q5

6. (a) Present the stiffness equations for a 1D bar element in matrix form. [5 marks]

The pin-jointed framework ABC is subjected to an external load as shown in Figure Q6. If each member has a length, L, of 1m and a value of the product AE of 200MN [n.b. A = cross-sectional area of a member and E = Young's modulus of the member material]

(b) Construct the stiffness matrix of the structure [10 marks]

If the applied load, F, is 20kN

(c) Determine the horizontal and vertical displacements at point B. [4 marks]

(d) Determine the reaction forces at points A and C. [6 marks]

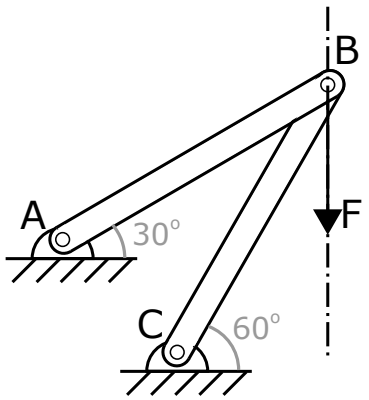

Figure Q6

The stiffness matrix of a truss element is:

$$[k_e] = \left(\frac{AE}{L}\right)\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta & -\cos^2\theta & -\cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & -\cos\theta\sin\theta & -\sin^2\theta \\ -\cos^2\theta & -\cos\theta\sin\theta & \cos^2\theta & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & -\sin^2\theta & \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

END